Turbulence modelling for the Shallow Water Equations

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Why Turbulence?

Turbulence causes:
• almost infinite flow detail
• momentum diffusivity

da Vinci, Heisenberg and Lamb all thought turbulence important!
Current trends in hydraulic modelling

Very coarse mesh
Possibly low-order spatial interpolation
Eddy viscosity model has low impact on results

Mesh resolutions commonly less than water depth
Higher-order spatial interpolation
Eddy viscosity model has high impact on results
Scale of Turbulence

• Discretised models of fluids required a turbulence closure model
• Boussinesq proposed to replace Reynolds stresses with turbulent eddy viscosity, $\nu_t$
• Prandtl proposed a length scale, $l_m$
• Length scale evolves
• In unconfined 3D turbulence exhibits “energy cascade”
• Two common modelling approaches: RAS and LES
Reynolds Average Stress (RAS) Turbulence Closure

Solutions spatially smooth

Excellent for steady state solutions or slowly varying in time
Large Eddy Simulation (LES) Turbulence Closure

- Solutions spatially detailed
- Excellent for transient solutions
- Statistical analysis of transient results sometimes needed
Turbulence in shallow fluid flows

In shallow fluid flows we have both 2D and 3D flow behaviour.

Energy cascade bi-modal

Bed friction converts larger scale 2D turbulence into smaller scale 3D turbulence.

Possible minimum in PSD at scales similar to depth.

I am about to present …

Three turbulence models

Three benchmark test cases (range of physical scales)

Determine optimum turbulence model parameters for each test case

Summary – is there a ‘one size fits all’ turbulence model?
Constant Viscosity Model

\[ \nu_t = C \]
Smagorinsky Turbulence Model

\[ \nu_t = M \Delta x \Delta y |S_{2D}| \]

\[ |S_{2D}| = \sqrt{\left( \frac{du}{dx} \right)^2 + \left( \frac{dv}{dy} \right)^2 + \frac{1}{2} \left( \frac{dv}{dx} + \frac{du}{dy} \right)^2} \]

Model is “diagnostic”
Wu Turbulence Model

\[ \nu_t = \sqrt{\nu_{2D}^2 + \nu_{3D}^2} \]

\[ \nu_{2D} = M_{2D} l_m^2 |S_{2D}| \]

\[ \nu_{3D} = M_{3D} l_m U^* \]

\[ U^* = \frac{\tau_{bed}}{\sqrt{\rho}} = \frac{|U| n \sqrt{g}}{h^6} \]

Model is “diagnostic”

\[ l_m = \min(h, y_{bank}) \]
Prandtl Turbulence Model

\[ \nu_t = Ml_m \sqrt{k} \]

\[
\frac{\partial hk}{\partial t} + u \frac{\partial hk}{\partial x} + v \frac{\partial hk}{\partial y} = \frac{\partial}{\partial x} \left( \nu_t \frac{\partial hk}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu_t \frac{\partial hk}{\partial y} \right) + h \left( P_k - C_D \frac{k^3}{Ml_m} \right)
\]

- Advection
- Diffusion
- Generation/Decay

\[ hP_k = h\nu_t |S_{2D}|^2 + |U|^2 \]

Model is “prognostic” with one additional field “k”
Test Cases

- Right angled flume bend, scale 0.15 m
- UK EA T06, scale 3 m
- Brisbane river historical flood event, scale 200 m
Kansas Uni right angled bend flume test
(15 cm wide rectangular section)
Kansas Uni flume test bend results


- Excellent correlation between head loss and upstream velocity head

- 90 deg bend loss factor 1.22-1.42
90 Bend Head Loss vs Mesh size

Mesh size converge

Optimum constant viscosity
\( \sim C = 0.004 - 0.005 \text{ m}^2/\text{s} \)
Sesame Street Game
90 Bend Head Loss vs Mesh size

Constant

Smagorinsky

Prandtl

Wu 2D

Wu 3D
UK EA T06 (Dam Break)

- Laboratory scale (~ 3m)
- Highly transient event
- Supercritical flow
- Hydraulic jumps
Gauge Data (first three gauges)
UKEA T06 Error vs Mesh Size

Constant

Smagorinsky

Prandtl

Wu 2D

Wu 3D
Meandering River – section of the Brisbane River

- D/S water level 2.7m
- U/S Q = 9,000 m³/s
- Steady flow model
- Peak of calibrated flow event
- Undulating bathymetry
- 20 to 30 m deep
- \( V_{\text{ave}} \) 3 to 4 m/s
- **Significant fraction of head loss due to eddy viscosity**
Brisbane River Head Loss vs Mesh Size

Smagorinsky

- $M = 0.2$
- $M = 0.5$
- $M = 1.0$

Recorded head loss 4m
Brisbane River Head Loss vs Mesh Size

Constant

Smagorinsky

Prandtl

Wu 2D

Wu 3D
## Optimum Parameters

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<thead>
<tr>
<th>Case</th>
<th>Constant</th>
<th>Smagorinsky</th>
<th>Wu 2D</th>
<th>Wu 3D</th>
<th>Prandtl</th>
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</tbody>
</table>

- Big variation
- Requires more memory

\[
C = \frac{\frac{1}{h^6}}{n}
\]

\[
D_{xx} = k_l |U| n \sqrt{g} h^5
\]

\[
\nu_{3D} = M \frac{|U| n \sqrt{g}}{h^6}
\]

So … should we have longitudinal and transverse eddy viscosity?
Summary

Constant eddy viscosity model requires scale-dependent parameter

Smagorinsky eddy viscosity model does not demonstrate mesh-size convergence

Wu 2D, Wu 3D, Prandtl models all performed well

Wu 3D showed best promise as computationally efficient and ‘one size fits all’

As ever, modellers encouraged to calibrate where possible, and to check mesh-size sensitivities